Conference on Stark's Conjectures and Related Topics

Talks by Invited Speakers

Speaker: Werner Bley, University of Augsburg

Title: 'Explicit Units and the Equivariant Tamagawa Number Conjecture'

Abstract: Let L/K be a finite abelian extension of number fields of group G. Let $T\Omega(L/K)$ denote the equivariant Tamagawa number of $h^0(\text{Spec } L)$. For a large class of extensions L/K the conjectural vanishing of $T\Omega(L/K)$ is equivalent to the existence of S-units of L satisfying a variety of explicit conditions. We explain this result and use it to produce numerical evidence for the equivariant Tamagawa number conjecture for cyclic extensions of real quadratic fields of prime degree.

Speaker: David Burns, King's College London

Title: T.B.A.

Abstract: T.B.A.

Speaker: Henri Darmon , McGill University

Title: T.B.A.

Abstract: T.B.A.

Speaker: David Dummit, U. Vermont at Burlington

Title: Computations Related to Stark's Conjecture

Abstract: The purpose of this talk will be to describe some of the numerical computations that have been performed related to Stark's Conjecture. These computations have provided additional supporting evidence in both the abelian and nonabelian cases of Stark's and related conjectures and have found application to the explicit construction of class fields.

Speaker: Matthias Flach, Caltech

Title: On the Equivariant Tamagawa Number Conjecture

Abstract: We give an introduction to the conjecture in the title which was formulated by David Burns and the speaker. It unifies the conjectures on special values of L-functions due to Bloch, Kato, Fontaine and Perrin-Riou on the one hand, and conjectures in Galois Module theory due to Chinburg, Ritter, Weiss et al on the other. We also mention some evidence for the conjecture.

Speaker: Cornelius Greither, U. der Bundeswehr, Munich

Title: Fitting ideals and the Brumer-Stark conjecture

Abstract: There is a renewed interest in Fitting ideals of arithmetic objects, at finite level (objects attached to number fields) and at infinite level (Iwasawa modules). After some remarks about results at finite level (joint work with D. Burns on the Coates-Sinnott conjecture), we turn to the situation at the infinite level. As soon as an action of a finite Galois group G is given besides the usual Λ-module structure, the standard Iwasawa modules tend not to have finite projective dimension over $\Lambda[G]$. This is a technical obstacle which may be circumvented by considering somewhat larger modules that take ramification into account; somehow this modification has to be removed again. The talk will discuss this method and its results for two typical Iwasawa modules; there appears to be a close relation to results of Kurihara. An application towards the Brumer-Stark conjecture will be presented.

Speaker: Dick Gross, Harvard U.

Title: 'Salem numbers and automorphisms of even, unimodular lattices'

Abstract: Salem numbers (or more generally, roots of monic reciprocal polynomials) are special cases of Stark units. We will review this connection, then discuss which reciprocal polynomials occur as the characteristic polynomials of automorphisms of even, unimodular lattices.

Speaker: David Hayes, U. Massachusetts at Amherst

Title: T.B.A.

Abstract: T.B.A.

Speaker: Masato Kurihara, Tokyo Metropolitan University

Title: 'On the structure of class groups of CM fields'

Abstract: I will talk about a generalization of a theorem by Kolyvagin and Rubin that the structure of the minus part of the ideal class group of a certain imaginary abelian field is determined by Stickelberger elements, to CM fields. I will also discuss the Fitting ideal of the Iwasawa module over the cyclotomic $\mathbb{Z}_p$ extension of a CM field.

Speaker: Cristian Popescu, Johns Hopkins University

Title: 'Rubin's integral version of the abelian Stark Conjecture'

Abstract: We will state Rubin's integral version of Stark's Conjecture for abelian L-functions of arbitrary order of vanishing at $s = 0$. We will provide evidence supporting this statement and, time permitting, will discuss some of its consequences.

Speaker: Xavier Roblot, IGD-U. Claude Bernard (Lyon)

Title: Numerical Verification of the Stark-Chinburg Conjecture for Some Icosahedral Representations

Abstract: Let ρ be an odd irreducible two-dimensional Galois representation such that its Artin L-function $L(s, \rho)$ has a first-order zero at $s = 0$. The Stark-Chinburg conjecture relates the value of the first derivative $L'(0, \rho)$ with some (conjectural) real unit living in the field fixed by the kernel of ρ. Irreducible two-dimensional representations are classified according to the isomorphism type of their images in $\mathrm{PGL}_2(\mathbb{C})$, the four possible types being dihedral, tetrahedral (A_4), octahedral (S_4), and icosahedral (A_5). Stark has provided illuminating examples in the dihedral cases; Chinburg has confirmed the conjecture numerically for five tetrahedral representations; and Fogel has confirmed it numerically for eight octahedral representations. In this talk, I will explain our numerical confirmation of the Stark-Chinburg conjecture for fourteen icosahedral representations.

Speaker: Karl Rubin, Stanford University

Title: Kolyvagin systems

Abstract: Since their introduction by Kolyvagin, Euler systems have been used in several important applications in arithmetic algebraic geometry. Kolyvagin's machinery proceeds in two steps. The first step takes as input an Euler system and gives as output a new collection of "derivative" cohomology

classes. The second step uses this system of derivative classes to obtain an upper bound on the size of an appropriate Selmer group.

Kolyvagin's derivative classes satisfy stronger interrelations than have previously been recognized. We call a system satisfying these strong interrelations a Kolyvagin system. The extra relations provide a rigid structure which resembles the leading term of an L-function. The extra rigidity also makes it possible to describe (under nice hypotheses) the size of the group of Kolyvagin systems attached to a given representation, and in particular to say when it is nonzero.

This talk is a report on joint work with Barry Mazur.

Speaker: Victor Snaith, University of Southampton

Title: 'Stark's conjecture and new Stickelberger phenomena'

Abstract: We introduce a new conjecture concerning the construction of elements in the annihilator ideal associated to a Galois action in the higher-dimensional algebraic K-groups of rings of integers in number fields. Our conjecture is motivic in the sense that it involves the (transcendental) Borel regulator as well as being related to l-adic étale cohomology. In addition, the conjecture generalises the well-known Coates-Sinott conjecture. By way of supporting evidence, we prove the corresponding (sometimes equivalent) conjecture for the Galois action on the étale cohomology of the cyclotomic extensions of the rationals.

Speaker: David Solomon, King's College London

Title: 'The p-adic Stark Conjecture at $s = 1$ and $\mathbb{Z}_p$-Extensions'

Abstract: We review a reformulation of Stark's conjecture at $s = 1$ in terms of complex twisted zeta-functions and a group-ring-valued regulator. We state an analogous p-adic conjecture and consider its behaviour as K varies in a $\mathbb{Z}_p$-tower. If the solutions exhibit certain properties (which imply in particular the boundedness of their denominators) then they must also satisfy a new and explicit p-adic conjecture at $s = m$ for each $m \in \mathbb{Z}$. We shall discuss these conditions and, if time allows, the resultant conjectures, particularly at $s = 0$.

Speaker: Harold Stark, U. California at San Diego

Title: 'Calculating values of p-adic Dirichlet series'

Abstract: The goal is to produce many p-adic digits of derivatives at zero.

Speaker: John Tate, U. Texas at Austin

Title: T.B.A.

Abstract: T.B.A.

Talks by Contributing Speakers

Speaker: Bisi Agboola, U. California at Santa Barbara

Title: 'On the anticyclotomic Iwasawa theory of CM elliptic curves'

Abstract: I shall describe how one may formulate, and partially prove an anticylotomic main conjecture for CM elliptic curves in the so-called 'exceptional case'. This conjecture is analagous to an anticyclotomic main conjecture for modules of Heegner points attached to non-CM elliptic curves formulated by B. Perrin-Riou and partially proved by M. Bertolini.

Speaker: Jean-Robert Belliard, U. de Franche Comté (Besançon)

Title: 'A Real Complement to Stickelberger's Theorem'

Abstract: Let k be a number field and K/k a finite abelian extension. Brumer-Stark's conjecture predicts the existence of explicit annihilators of the class group of K, as well as other properties involving class field theory. When the base field k is $\mathbb{Q}$ this conjecture holds true as a consequence of classical Stickelberger's theorem. However if the abelian number field K is totally real, "trivial zeroes" of L functions forces the explicit annihilator to be zero. Hence in these cases complements are needed. In the p-adic context David Solomon constructed these previously missing totally real p-adic Gauß sums, and conjectured the analogous annihilation of real classes. I will present an alternative approach to this construction and some applications obtained in a joint work with Thong Nguyen Quang Do.

Speaker: Jason Colwell, Caltech

Title: 'Elliptic curves with CM by nonmaximal orders'

Abstract: We indicate a proof of the equivariant Tamagawa number con-

jecture of Burns and Flach for elliptic curves with complex multiplication by nonmaximal orders in the CM-field.

Speaker: Udi de Shalit, Hebrew U.

Title: 'Some remarks on Stark's conjecture in abelian extensions of CM fields'

Abstract: Besides abelian extensions of Q or quadratic imaginary fields, the only other case where one knows how to generate explicitly abelian extensions of number fields in a systematic way, are the towers of CM fields obtained by adjoining torsion points of abelian varieties with CM. Contrary to the first two examples, there is no explicit construction of units in these abelian extensions. Yet, the p-adic L functions, which are a manifestation of the Euler system of cyclotomic or elliptic units in the first two examples, were constructed by Katz and by Hida and Tilouine, by other means.

In this talk we shall discuss the conjectural relation that exists between Euler systems of Stark elements in these towers and p-adic L functions, and in particular the surprising role of the 'p-adic period relations'.

Some of the results are old (unpublished) work, newly reformulated in light of recent papers of Rubin. Some of the remarks are new.

Speaker: Karrolyne Fogel, California Lutheran U.

Title: 'Stark's Conjecture for Octahedral Extensions'

Abstract: Stark's Conjecture involves the construction of units in an algebraic field from the Artin L-functions associated with representations on the Galois group of the field extension. We investigate the conjecture for non-abelian extensions with $\mathrm{GL}_2(\mathbb{F}_3)$ as the Galois group. Seven examples supporting the conjecture were constructed by adjoining points of order 3 on an elliptic curve to the field of rational numbers. The projective image of each of the representations arising from these field extensions is octahedral. This work extends that of Chinburg for tetrahedral representations.

Speaker: Paul Gunnells, U. Massachusetts at Amherst

Title: 'Cusp singularities and special values of L-functions'

Abstract: Let F be a totally real number field of degree n, let $M \subset F$ be a $\mathbb{Z}$-module of rank n, and let V be a finite index subgroup of the totally

positive units of F satisfying $V \cdot M = M$. One can attach to the pair (M, V) two objects: first, a set of Dirichlet series $L(M, V; k) = \sum N(\mu)^{-k}$, where $k = 1, 2, 3, \ldots$, and the summation is taken over nonzero orbits of V in M; and second, a cusp singularity X of a Hilbert modular variety. In the 70's Hirzebruch conjectured that the special value $L(M, V; 1)$ could be computed in terms of intersection numbers of the exceptional divisor of a resolution of X. This was proved independently by Atiyah-Donnelly-Singer and Müller in the mid 80's, using heavy analytic machinery.

More recently, Satake proposed a similar interpretation of the special values for $k > 1$, in terms of Hilbert-Picard modular cusps. We will describe a proof of Satake's conjecture that avoids heavy machinery and explicitly shows how the intersection numbers contribute to the special values. In particular this affords a geometric interpretation of the special values at nonpositive integers of the partial zeta functions attached to F. We will also describe work in progress on Satake's full conjecture, which connects resolutions of general arithmetic cusps with certain zeta functions.

This is joint work with R. Sczech and J. Sturm.

Speaker: Anthony Hayward, King's College London

Title: 'On refined class number formulas for higher derivatives of L-series'

Abstract: In 1987 Gross conjectured certain rather striking congruence relations for the values of abelian L-functions at s = 0. In a seemingly different direction, in 1996 Rubin formulated a natural integral refinement of Stark's Conjecture for the values of higher derivatives of abelian L-series at $s = 0$. We shall discuss a common refinement of these conjectures, due to Burns, which is motivated by, and in many cases a consequence of, the equivariant Tamagawa number conjecture. We give some classes of extensions for which there is evidence in favour of this. We then show that the analogue of Gross's Conjecture for first derivatives of certain L-series, formulated by Darmon in 1995, can be interpreted in terms of a natural "base-change" property in the new framework.

Speaker: Jeff Hooper, Acadia U.

Title: 'Chinburg Invariants and Generalized Quaternion Groups'

Abstract: In this talk we shall present new results regarding the second Chinburg conjecture for some families of generalized Quaternion groups.

The conjecture is known to hold in the absolute case for all tamely ramified extensions and for most abelian extensions of $\mathbb{Q}$, by work of Taylor and Greither.

For extensions which are wild and nonabelian, the result is known for quaternion groups of order 8 (H.-Snaith-Tran), but little is known for more general cases.

This is joint work with Steve Wilson.

Speaker: Manfred Kolster, McMaster U.

Title: 'Motivic formulation of the Lichtenbaum Conjecture'

Abstract: The talk will survey the state of the art of the Lichtenbaum Conjecture and in particular concentrate on the contributions of the prime 2 to motivate a motivic reformulation of the Conjecture.

Speaker: Joongul Lee, KIAS

Title: 'Refined class number formula for function fields'

Abstract: Let L/K be an abelian extension of function fields of one variable over finite field. If $[L : K] = l^m$ for some prime l, we prove a conjecture of Gross regarding the Stickelberger element under the condition that K has no nontrivial l-th root of 1 and that the class number of K is prime to l.

Speaker: Thong Nguyen Quang Do, U. de Franche Comté (Besançon)

Title: 'Equivariant Main Conjecture and Fitting Ideals'

Abstract: For an abelian CM extension of a totally real field, we propose (and prove, assuming the vanishing of the μ-invariant),an equivariant version of Iwasawa's Main Conjecture in terms of Fitting ideals. This EMC is based on previous work of Ritter and Weiss. We apply it to study the conjectures of Coates-Sinnott, and of Brumer.

Speaker: Michael Reid, U. of Arizona

Title: 'On the p-adic Stark conjecture at a real place'

Abstract: The p-adic Stark conjecture was put forth by Gross. Its intent is to understand the Stark unit (more precisely, a modification of the usual Stark unit) from a p-adic analytic viewpoint. Dummit and Hayes noticed,

while calculating some examples of Stark units in the situation where the split place is a real place, that the resulting Stark units were often squares. they subsequently showed that this follows as a consequence of the p-adic Stark conjecture.

We show that the phenomenon discovered by Dummit and Hayes should hold in much greater generality. the significance of this, as already noted by Dummit and Hayes, is that the abelian part of Stark's conjecture holds for free in such instances.

Speaker: Robert Sczech, Rutgers U.

Title: 'A refinement of Stark's conjecture over complex cubic fields'

Abstract: According to the conjecture of Stark, the first derivative of a partial zeta function at $s = 0$ yields (the logarithm of) the absolute value of an algebraic integer. If the underlying base field is not totally real, that information can not be used to recover the complex number (Stark unit) inside the absolute value sign. In our talk we report on a conjectural formula (up to a high order root of unity) for the Stark unit inside the absolute value sign in the case of a complex cubic base field.

Speaker: Romyar Sharifi, Harvard U.

Title: ' On a cup product and local embeddings of p-units'

Abstract: I will report on joint work with W. McCallum in which we consider a cup product in the Galois cohomology of a number field K with restricted ramification set S, more precisely, on H^1 with μ_n-coefficients. The cup product can be restricted to pairs of S-units, and it induces the negative of the natural map from Milnor K_2 to algebraic K_2 (modulo n) of S-integers.

In the case that p is a prime satisfying Vandiver's conjecture, $K = \mathbf{Q}(\zeta_p)$, $n = p$ and S consists of the unique prime above p, we conjecture that the latter map is a surjection. We have a conjectural calculation of the pairing on p-units for all $p < 10{,}000$. Furthermore, we have exhibited surjectivity for $p = 37$. This is done by computation, the principle being that one can relate the pairing to norm residue symbols on the p-Hilbert class field H of K. The problem then reduces to the study of the local embeddings of the p-units of H at the p primes of H above p.

Speaker: Brett Tangedal, College of Charleston

Title: ‘ Resolvents constructed from Stark units’

Abstract: Lagrange resolvents are constructed from Stark units and some of their properties, including a Stickelberger type behavior, are considered.