Spectral properties of evaluation maps and Diophantine geometry (Conference)

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Let $X \hookrightarrow \mathbf{P}^N_K$ be a projective variety over a number field K, and $\hat{V} = \lim_{\rightarrow} V_i$ a formal subscheme of X defined by an inductive sytem of closed subschemes V_i of X. To this data we may associate the evaluation maps

$$\eta_{i,D} : \Gamma(X, \mathcal{O}(D)) \longrightarrow \Gamma(V_i, \mathcal{O}(D))$$

mapping the global sections s in $\Gamma(X, \mathcal{O}(D))$ to their restrictions $s_{|V_i}$.

It turns out that a crucial point of many proofs in Diophantine geometry is played by the spectral properties of such evaluations maps, over the local fields K_v completions of K. Namely, under suitable geometric conditions, the characteristic values of $\eta_{i,D|K_v}$ decay "very fast".

In my talk, I plan to explain this in some detail, and to speculate on some possible related links between non-commutative geometry and diophantine geometry.

Theta series and dimension over $\mathbf{F}_1$ of spaces of sections of hermitian vector bundles (Workshop)

It is "well-known" that the logarithm of the theta series

$$\log \sum_{v \in E} e^{-\|v\|^2}$$

associated to some euclidean lattice $\overline{E} := (E, \|.\|)$ plays the role of the dimension over $\mathbf{F}_1$ of the elusive space of "regular sections" over $\operatorname{Spec} \mathbf{Z}$ compactified (by means of the archimedean absolute value on $\mathbf{Z}$) of the hermitian vector bundle defined by $\overline{E}$. This has been recently emphasized by Roessler, and Schoof and van der Geer.

I plan to discuss the origin of this analogy (namely the comparison of the classical works of Hecke and F.K. Schmidt about the zeta functions of number fields and function fields), and its relevance in the proof of “concrete” results in geometry of numbers and Diophantine geometry.