

Recent Developments in Higher Dimensional Algebraic Geometry
Japanese-American Mathematics Institute 2006
Johns Hopkins University

Marco Andreatta - On Fano manifolds

I present some recent results on Fano manifolds connected to a conjecture of S. Mukai and to classification problems.

Caucher Birkar - mld's vs thresholds and flips

(Joint work with Shokurov) I will discuss the relation between ACC for mld's, ACC for a-lc thresholds and lc thresholds, ACC for type orders, boundedness of Fano's, termination of log flips and existence of log flips.

Alexey Bondal - Integrable systems related to triangulated categories

We give a general overview of the homotopy origin of triangulated categories. Then we describe two useful tools of studying triangulated categories - Grothendieck group and the theory of exceptional collections. Then we show how the study of numerical invariants of triangulated categories with exceptional collections lead to some integrable systems, which could be explored by means of algebraic geometry.

Andrei Căldăraru - The Hopf algebra governing orbifold Hochschild cohomology

When X is a quasi-projective variety, Kapranov argued that the object $T_X[-1]$ in the derived category of X is a Lie algebra object, whose universal enveloping algebra governs the behavior of the Hochschild cohomology ring of X . We discuss the construction of a similar Lie algebra object for global quotient orbifolds.

Frédéric Campana - Hyperbolicity of some weakly special, but nonspecial threefolds.

A complex projective manifold X is said to be weakly special if none of its unramified covers maps rationally onto a variety of general type. It is said to be special if it has no map onto an orbifold of general type. Being special implies being weakly special, the converse holding for curves and surfaces, but no longer for threefolds, as shown by examples of Bogomolov-Tschinkel. Special manifolds are conjectured (math AG/0110051, also Ann. Inst. Fourier 04) to be precisely the ones for which the Kobayashi pseudometric vanishes identically. In a joint work with M. Paun, we show that certain of the Bogomolov-Tschinkel threefolds enjoy indeed exactly the conjectured hyperbolicity properties. For these we use integral parts of orbifold differential operators, as substitute for differential operators on virtual ramified covers. The arithmetic analogue of these hyperbolicity properties remain open. Interestingly, they conflict with a conjecture by Abramovitch and Colliot-Thelene.

Fabrizio Catanese - Rigid varieties, the action of the absolute Galois group on moduli spaces, and the QED classification

I will report on work, recent or in progress, joint with Ingrid Bauer, resp. Fritz Grunewald, resp. Soenke Rollenske. I will first explain the construction of some simple varieties, which are obtained from the actions of a group G on some algebraic curves. The first examples are varieties isogenous to a product, and Beauville surfaces. Another interesting class is the class of double étale Kodaira fibrations, for which I will discuss the slope and the packing question. It turns out that the moduli space of the above varieties is given by a product

of moduli spaces for pairs (C,G) , and we obtain thus strongly rigid or rigid examples in case where (C,G) is a triangle curve. The absolute Galois group acts on moduli spaces, in particular on the points corresponding to rigid varieties, and I will show that in the above cases this action is essentially equivalent to the action on Grothendieck's dessins d'enfants. The action here is visible as an action which changes the fundamental groups of the above manifolds, so the class of children drawings is replaced by a certain class of groups (this phenomenon that the topological type of a variety is not Galois invariant was observed long ago by J.P. Serre). I will finally describe some Kuga-Shavel rigid surfaces, constructed from quaternion algebras, which are interesting concerning the Q.E.D. classification of algebraic varieties (i.e., the classification modulo the equivalence generated by birational equivalence, deformation equivalence, and quasi-étale equivalence).

Ivan Cheltsov - Finite subsets of projective spaces and topology of threefolds.

We study properties of finite subsets of projective spaces in order to apply them to the factoriality problem of nodal threefolds. In particular, we prove that the points of a finite subset $\Sigma \subset \mathbb{P}^n$ impose independent linear conditions on homogeneous forms on \mathbb{P}^n of degree $\lfloor n\lambda/2 - n \rfloor$ in the case when at most λk points of the set Σ lie on a curve in \mathbb{P}^n of degree k and $|\Sigma| < \lambda \lfloor \lambda/2 \rfloor$, where $\lambda \in \mathbb{N}$ and $n \geq 3$. As a corollary we obtain the factoriality of a double cover of \mathbb{P}^3 branched over a nodal surface in \mathbb{P}^3 of degree $2r$ having less than $(2r - 1)r$ singular points, which is a sharp result, because there are non-factorial double covers of \mathbb{P}^3 branched over a nodal surface of degree $2r$ with $(2r - 1)r$ singular points. See <http://xxx.arxiv.org/abs/math.AG/0511578>.

Ciro Ciliberto - On the Andreotti–Mayer loci in \mathcal{A}_g .

The Andreotti–Mayer locus $N_{g,k}$ in \mathcal{A}_g is the set of all ppav (A, Θ) such that Θ has a singular locus of dimension at least k . These loci have been considered by several authors (Andreotti–Mayer, Mumford, Beauville, Debarre, Smith–Varley et al.) since they naturally appear in several instances of the geometry of ppav. In this talk I will report on (partly in progress) work in collaboration with G. van der Geer on estimates on the dimension of components of $N_{g,k}$. In particular we have a conjecture which says that an irreducible component N of $N_{g,k}$ has codimension at least $\binom{k+2}{2}$ in \mathcal{A}_g and equality holds if and only if either $g = k + 4$ or $g = k + 3$ and N is the jacobian or the hyperelliptic locus. I will outline a proof of this conjecture for $k = 1$.

Alessio Corti - On the quantum cohomology of Fano 3-folds

I aim to study the quantum cohomology of Fano 3-folds from first principles. Eventually I hope this can be used to give a new proof of properties of the classification, such as the genus bound. This will be an accessible talk describing ideas and work in progress.

Olivier Debarre - Singularities of divisors on abelian varieties

This is joint work with Ch. Hacon. Building on previous work of Kollár, Ein, Lazarsfeld, and Hacon, we show that ample divisors of low degree on a complex abelian variety have mild singularities.

Lawrence Ein - Applications of spaces of arcs to birational geometry

Tommaso de Fernex - Adjunction beyond thresholds and birationally rigid hypersurfaces.

We give an affirmative answer to a conjecture of Pukhlikov, proving that for N greater or equal to 4, all smooth hypersurfaces of degree N in the N -dimensional projective space are birationally superrigid. The case $N=4$ of this result is the celebrated theorem of Iskovskikh and Manin, which in fact started this whole direction of research; other cases were established in papers by Pukhlikov, Cheltsov, and Ein, Mustata and myself. The main new ingredient to obtain the complete result is an adjunction formula for singularities of pairs under restriction that, under suitable conditions, generalizes to higher codimensions the well-known formula for hyperplane sections derived from Shokurov's connectedness principle.

Osamu Fujino - A transcendental approach to Kollár's injectivity theorem

We treat Kollár's injectivity theorem from the analytic (or complex differential geometric) viewpoint. More Precisely, we give a curvature condition which implies Kollár type cohomology injectivity theorems.

Paul Hacking - Compactifications of moduli spaces of del Pezzo surfaces

We describe nice compactifications of the moduli spaces of smooth del Pezzo surfaces analogous to the compactification of the moduli space of n -tuples of distinct points on the projective line given by the moduli space of stable curves of genus zero with n marked points. Joint work with Sean Keel and Jenia Tevelev.

Brendan Hassett - Towards a canonical model for the moduli space of curves

This is joint with D. Hyeon. Consider the moduli space of stable curves as a log-variety, with boundary δ corresponding to the nodal curves. We seek to describe its log canonical model with respect to $K + A\delta$. When $A = 1$, we recover the moduli space of stable curves; for $A = 0$, this would be the canonical model of the moduli space, which is expected to exist for $g > 22$ after work of Eisenbud-Harris-Mumford and Farkas. For intermediate values of A , the log canonical model can be constructed with Geometric Invariant Theory. As A decreases, the log canonical model parametrizes curves with increasingly complicated singularities: cusps, tacnodes, and worse.

Akira Ishii - On the derived category for the minimal resolution of A_n singularities

Shihoko Ishii - The Nash problem for a toric pair

The Nash problem asks if the set of essential divisors of a variety X corresponds bijectively to the set of irreducible components of the closed subset $\pi^{-1}(\text{sing}X)$ in the arc space X_∞ . This problem is affirmatively answered for a toric variety, while negatively answered in general. So it is natural to expect that the set of essential divisors of a toric pair (X, \mathfrak{a}) corresponds bijectively to the set of irreducible components of a reasonable closed subset in the arc space.

In this talk I will show that the set of essential divisors of a toric pair (X, \mathfrak{a}) consisting of a \mathbb{Q} -factorial toric variety X and an invariant ideal \mathfrak{a} corresponds bijectively to the set of irreducible components of the closed subset $\pi_V^{-1}(Z(\mathfrak{a}O_V) \cup \text{sing}V) \subset V_\infty$, where $V \rightarrow X$ is an equivariant morphism corresponding to a dual subdivision with respect to the Newton polygon of \mathfrak{a} . In particular, if \mathfrak{a} is invertible, this closed subset is $\pi_X^{-1}(Z(\mathfrak{a}) \cup \text{sing}X) \subset X_\infty$.

Masayuki Kawakita - On inversion of adjunction

Recently Hacon and McKernan showed the existence of flips on the assumption of the minimal model program in lower dimension, based on the idea of a proof of the existence of four-fold flips due to Shokurov. Their work reduced the completion of the minimal model program to the termination of flips, and the termination follows from the two conjectures on minimal discrepancies (lower semi-continuity and ascending chain condition). Hence it is desirable to investigate singularities from the point of view of minimal discrepancies. I have begun to research them by showing inversion of adjunction on log canonicity. It states that for a log pair $(X, S+B)$ with S reduced, $(X, S+B)$ is log canonical if and only if the induced pair (S', B') on the normalisation of S is log canonical. I will explain inversion of adjunction and its related topics from the perspective of the termination of flips.

Yujiro Kawamata - Derived categories and minimal models

I will survey some results on the relationship between derived categories and minimal model program.

Stefan Kebekus - Families of Canonically polarized Varieties over Surfaces

(math.AG/0511378, with Sándor Kovács.) Shafarevich's well-known hyperbolicity conjecture asserts that a family of curves over a quasi-projective 1-dimensional base is isotrivial unless the logarithmic Kodaira dimension of the base is positive. More generally it has been conjectured by Viehweg that the base of a smooth family of canonically polarized varieties is of log general type if the family is of maximal variation. In this talk, we relate the variation of a family to the logarithmic Kodaira dimension of the base and give an affirmative answer to Viehweg's conjecture for families parametrized by surfaces.

János Kollár - A conjecture of Ax and degenerations of Fano varieties

James Ax conjectured that every pseudo algebraically closed field is C_1 . We prove this conjecture in characteristic 0 by relating it to degenerations of Fano varieties.

Sándor Kovács - Boundedness of families of canonically polarized manifolds

(Joint work with Max Lieblich). We show that the number of deformation types of canonically polarized manifolds over a smooth base variety is finite, and that this number is uniformly bounded in any finite type smooth family of base varieties. For families of curves (of genus at least two) this reduces to a conjecture of Shafarevich, of which the finiteness part was proven by Parshin and Arakelov while the uniformity part by Caporaso.

Robert Lazarsfeld - Asymptotic invariants of line bundles and convex bodies

While ample divisors on a projective variety satisfy many beautiful properties that render their behavior particularly tractable, it was traditionally believed that arbitrary effective divisors are mired in pathology. However it has recently become clear that a surprising amount of the classical picture goes through in general, provided that one works in an asymptotic setting. At the same time, many open questions remain.

In the first part of the talk I will give a general introduction to this circle of ideas, focusing on the volume of line bundles. This is an invariant that encodes the rate of growth of multiples of an effective divisor.

In the last few minutes, I will describe some work in progress with Mustata involving a construction, essentially due to Okounkov, that relates this invariant to the volume (in the classical sense) of a convex body.

Daisuke Matsushita - On existence conditions of Lagrangian fibrations

Let X be a projective irreducible symplectic manifold and L a nef divisor on X . Assume that the nef dimension of L is strictly less than the dimension of X . We prove that L is semiample. If there are rooms of time, we discuss another existence condition of Lagrangian fibrations.

James McKernan - A new approach to Mori theory.

We outline a new conjectural approach to proving some of the standard results of Mori theory, which instead of directly trying to establish existence and termination of flips, instead proceeds by trying to establish finite generated directly. The main new method is the method of multiplier ideals due to Siu and Kawamata, and Shokurov's saturation of an algebra.

Mircea Mustață - Spaces of arcs and singularities

Yoshinori Namikawa - Flops and Poisson deformations of symplectic varieties

We shall introduce a Poisson structure on a convex symplectic variety and discuss its Poisson deformation. As an application, we shall prove that the singularities are preserved under a symplectic flop.

Keiji Ogusio - Birational automorphism groups of hyperkähler manifolds

I would like to show first the following:

Theorem. Let G be a subgroup of the birational automorphism group $BirM$ of a projective hyperkaeher manifold. Then either (1) G is an almost abelian group of rank at most $\max(1, \rho(M) - 2)$, or (2) G contains a non-commutative free group $\mathbf{Z} * \mathbf{Z}$.

Next I would like to compare this with Mordell-Weil group of abelian fibered varieties (to construct examples).

Morihiko Saito - Bernstein-Sato polynomials of hyperplane arrangements

We explain a new method to calculate the b-function of a hyperplane arrangement using a solution of Aomoto's conjecture due to Esnault, Schechtman, Viehweg, and others.

Hiroshi Sato - On toric morphisms with anti-nef canonical divisors

Karen Smith - Interpreting jumping numbers for curves on smooth surfaces.

Let Y be a singular variety embedded in a smooth variety X . The jumping numbers of the pair (X, Y) are a sequence of positive rational numbers, starting with the log canonical threshold, that encode information about a log resolution of the pair. In particular, the sequence of jumping numbers gets more complicated as the number of exceptional divisors needed to resolve (X, Y) increases, or as the multiplicities along them in the ideal of Y get larger. On the other hand, some exceptional divisors never contribute to the jumping numbers. Obviously, they can happen if one makes some extraneous blowups, but some "essential" exceptional divisors can also be irrelevant from the point of view of jumping numbers. In this talk, we focus on identifying such irrelevant exceptional divisors in the case where Y is a curve on a smooth surface. In this case we show: an exceptional divisor E in

a minimal log resolution of X contributes to the jumping numbers of (X, Y) if and only if it has intersection multiplicity at least three with the rest of the reduced total transform of Y .

Jason Starr - Higher Chern classes and rational surfaces on varieties

I will present some results relating positivity of the second graded piece of the Chern character of X to existence of rational surfaces on X , much as the first Chern class relates to rational curves on X . I will also discuss some baby steps towards classifying Fano manifolds whose second graded Chern character is positive. This is joint work with A. J. de Jong and uses joint work with Coskun and Harris.

Hironmichi Takagi - On a moduli theoretic characterization of a \mathbb{Q} -Fano 3-fold of genus 6

A smooth prime Fano 3-fold of genus 6 is highly nonrational. But there is a rational prime \mathbb{Q} -Fano 3-fold X of genus 6 with one $1/2(1,1,1)$ -singularity. X admit an explicit birational map similar to Fano's double projection of smooth prime Fano 3-folds. More precisely, X is birational to the blow-up of a smooth quadric 3-fold along a smooth curve C of genus 6. I discuss the way how to reconstruct X from the data on C . This work is inspired by a Brill-Noether theoretic characterization of smooth Fano 3-folds of genus seven or nine due to Mukai.

Shunsuke Takagi - A characterization of plt singularities via Frobenius splitting.

We prove that a log pair (X, Δ) is plt if and only if reduction to characteristic $p \gg 0$ of (X, Δ) is a divisorially F -regular pair which is a class of singularities in positive characteristic defined in terms of Frobenius splitting.

Yukinobu Toda - Stability conditions and crepant small resolutions

The space of stability conditions for a triangulated category was constructed by T. Bridgeland to give a mathematical framework for the Douglas' work on Π -stability. I give the description of this space for a triangulated category associated to three dimensional crepant small resolutions. The resulting space has two chamber structures, one of them is geometric and the other is algebraic.

Keiichi Watanabe - F -thresholds with applications to ring theoretic properties and multiplicity

Jarek Wisniewski - On varieties arising as models of phylogenetic trees

I would like to report on a joint work with Weronika Buczynska. The main result is as follows: geometric models of binary symmetric 3-valent phylogenetic trees with n leaves are Fano varieties of dimension $2n-3$ with Gorenstein terminal singularities and they are deformation equivalent in $P^{2^{n-1}-1}$.