

Supplementary Problems 4

- (1) Imagine a piece of graph paper. Starting at the origin, draw a path to the point $(10,10)$, that stays on the grid lines (which are one unit apart) and has a total length of 20. For example, one path is to go from $(0,0)$ to $(0,7)$ to $(4,7)$ to $(4,10)$ to $(10,10)$. Another path goes from $(0,0)$ to $(10,0)$ to $(10,10)$. How many possible different paths are there?
- (2) How many different ordered triples (a, b, c) of *positive* integers are there such that $a + b + c = 50$?
- (3) In how many ways can four squares, not all in the same row or column, be selected from an 8-by-8 chessboard to form a rectangle?
- (4) How many *strictly increasing* sequences of positive integers begin with 1 and end with 1000?
- (5) You can buy chicken nuggets only in boxes of a or b nuggets, where a and b are relatively prime. Call the integer N “edible” if it is possible to buy a meal of exactly N nuggets. Prove that every $N \geq (a - 1)(b - 1)$ is edible. Also, prove that, of the non-negative integers less than or equal to $(a - 1)(b - 1)$, exactly half are edible.